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Computation of Craze Surface Stresses in High Impact Polystyrene

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Summary

The finite element method is used to compute surface stresses for a craze in high impact polystyrene (HIPS) and the results are compared with those obtained using a distributed dislocation stress analysis. There is very good agreement between the stress profiles for the major portion of the craze length. The estimates differ in the craze tip region where calculated stresses are very sensitive to small changes in the displacement profile.

Introduction

A transmission electron microscopy technique (LAUTERWASSER and KRAMER 1979) has recently (DONALD and KRAMER 1982) been used to measure craze surface displacements in HIPS. Thin films of the polymer are bonded to a copper grid which is then strained until crazes nucleate at rubber particles. A distributed dislocation stress analysis (WANG and KRAMER 1982) is applied to the displacement profile in order to compute both the craze surface stress profile and the applied stress. The finite element method, which has been used to compute craze surface displacements and stresses (BEVAN 1981, 1982), is utilised to determine the stress profile which is then compared with that obtained by the distributed dislocation analysis.

Theoretical

The distributed dislocation method (BILBY and ESHELBY 1972) has been applied (WANG and KRAMER 1982) to plastic zones at crack tips. The analysis is applicable to narrow wedge-shaped zones such as those considered in the Dugdale model (DUGDALE 1960). It has been observed (BROWN and WARD 1973) that crazes in glassy polymers have this shape.

The dislocation density $\alpha(x)$ is, with unit Burgers vector, given by

$$\alpha(x) = -2 \frac{dv(x)}{dx} \qquad (1)$$

where v is the displacement of the surface of the plastic zone.

The requirement of no stress singularity at the crack tip governs the surface stress profile and enables the applied stress, σ_∞, to be calculated. The surface stress is

$$S(x) = \frac{E^* x (x^2 - c^2)^{\frac{1}{2}}}{2\pi} \int_c^a \frac{dx_1 \, a(x_1)}{(x_1^2 - c^2)^{\frac{1}{2}}} \cdot \frac{1}{(x^2 - x_1^2)} \qquad (2)$$

where $E^* = \begin{cases} E & \text{plane stress} \\ E/(1-\nu^2) & \text{plane strain} \end{cases}$

In the above equation, E is Young's Modulus, ν is Poisson's ratio, c is the half length of the crack and a is the total half crack plus craze length.

The applied stress equation is

$$\sigma_\infty = \frac{E^*}{2\pi} \int_c^a dx \; a(x) \; \frac{1}{(x^2 - c^2)^{\frac{1}{2}}} \qquad (3)$$

The method is not restricted to dislocation arrays since the surface stress profile required to maintain a given surface displacement profile will be determined by the elastic response of the surrounding matrix. The plastic zone can therefore be represented by an equivalent continuous array of dislocations which produces the same displacements.

Computation and Discussion.

The rubber particle is treated as a crack, of half length equal to the radius of the particle, when the dislocation density method is used in this application. In the plane stress finite element analysis the particle is modelled as a low modulus circular inclusion. Three dimensional analysis would require that the particle be represented by a sphere. However, three dimensional finite elements are expensive in computer time and a previous study (BEVAN 1981) has shown that two dimensional analysis is adequate for crazes. The stress concentration factor for a cylinder differs from that of a sphere but the difference in computed stresses for a measured displacement profile should be small except, possibly, for the vicinity of the particle tip. The finite element idealization of the region including the particle and the craze is shown in Figure 1. The remainder of the mesh is similar to that published recently (BEVAN 1982).

The applied stress, calculated using equation (3), and the experimental values of craze surface displacements are included in the finite element program input. The output gives the corresponding craze surface stress profile. It is found that almost identical stresses are obtained if the particle is treated as a circular void. Craze surface stress profiles are plotted in Figure 2 and it is seen that there is excellent agreement for most of the craze length between the finite element profile and that calculated by DONALD and KRAMER (1982) using equation (2). The modelling of the rubber

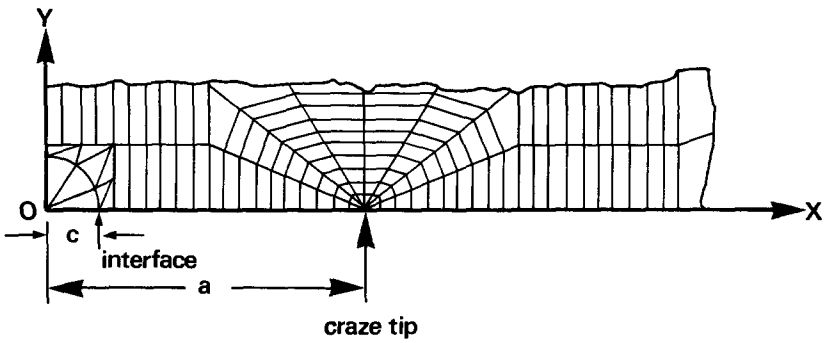

Figure 1. Portion of finite element mesh including rubber particle and the craze.

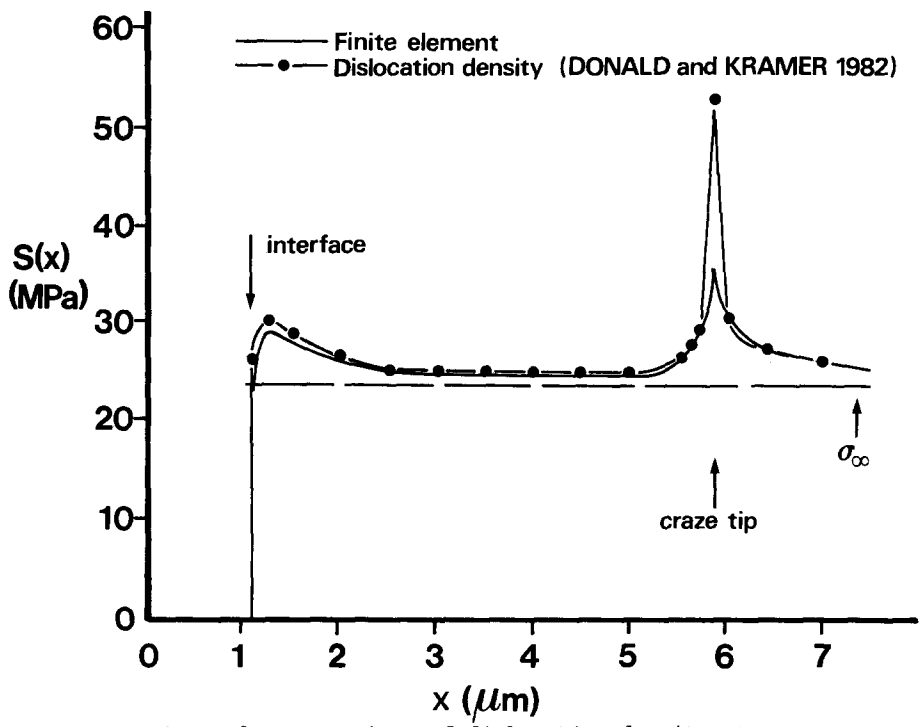

Figure 2. Comparison of dislocation density stress analysis and finite element method estimates of the craze surface stress profile.

particle as a crack therefore does not produce any appreciable error in the calculated stresses. This does not imply that the crack would produce the same tip displacements as the particle but it shows that the analysis may be used when the measured displacements have not been produced by a crack.

There is a marked difference in the two profiles as the craze tip is approached. It is known (ALLISON 1979) that finite element estimates of stress concentration factors are lower than experimental ones but the discrepancy in tip stresses is in this case due mainly to differences in craze tip displacement profiles. It is seen from equation (2) that the computed stress normal to the interface at any point is very sensitive to displacement derivatives in the immediate vicinity of the point. Thus the calculated surface stress at the craze tip is dependent on the precision of measurements of craze opening displacements near the tip. Displacement measurements are made up to about 0.25 μm from the craze tip. This resolution is very good for typical crazes and deformation zones with lengths in the region of 100 μm but for HIPS crazes it is necessary to assume the shape of the displacement profile in the last 5% or so of the craze length. The difference in craze tip stress profiles shown in Figure 2 reflects differences in the assumed shape of the displacement profiles for this region. If the same displacement profile is used in both computations, the stress profiles are much closer but the estimated stress concentration factor at the craze tip is at best an approximation. Tip stresses can be computed more accurately for longer crazes where finite element estimates of the stress concentration factor are in reasonable agreement with, although lower than, those obtained by the dislocation density method.

The present study has concentrated on the calculation of $S(x)$ and not the applied stress, σ_∞. The latter can readily be computed using the dislocation density stress analysis but this analysis does not take into account the lateral constraint imposed by the copper grid on the polymer film. An accurate estimate of the applied stress could be facilitated by measuring the displacements at the boundary of the grid and the film. The displacements and the boundary conditions of the experiment would then be included in the finite element program. The effect of lateral constraint is at present being considered and preliminary results suggest that the applied stress for craze nucleation and growth in HIPS is about 14% greater than that calculated assuming that the boundary displacements are produced by uniaxial tension.

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